Path Tracing
Path Tracing Overview

Partition the rendering equation

• Direct lighting – non-recursive
• Indirect lighting – recursive

Monte Carlo estimate for each partition separately

• Possible to take just one sample for each
• Assume: 100s - 1000s of paths sampled per pixel

Terminate paths with Russian Roulette
Partitioning the Rendering Equation

\[ L_o(p, \omega_o) = L_e(p, \omega_o) + \]

\[ \int_{H^2} f_r(\omega_i \to \omega_o) L_{o,d}(tr(p, \omega_i), -\omega_i) \cos \theta_i \, d\omega_i + \]

\[ \int_{H^2} f_r(\omega_i \to \omega_o) L_{o,i}(tr(p, \omega_i), -\omega_i) \cos \theta_i \, d\omega_i \]

Incident “direct” illumination: \( L_{o,d}(tr(p, \omega_i), -\omega_i) \)

- This term includes only emissive light (light sources)
- Estimate integral with usual importance sampling over light

Incident “indirect” illumination: \( L_{o,i}(tr(p, \omega_i), -\omega_i) \)

- This term includes only non-emissive light (inter-reflections)
- Estimate with recursive evaluation of rendering equation
1-Bounce Path Connecting Ray to Light
2-Bounce Path Connecting Ray to Light
3-Bounce Path Connecting Ray to Light

Camera

Light

Viewing window
Pixel
Traced ray
Viewpoint

A
B
C
Recursive Estimation of Indirect Illumination

\[
\int_{H^2} f_r(\omega_i \rightarrow \omega_o) L_{o,i}(tr(p, \omega_i), -\omega_i) \cos \theta_i \, d\omega_i
\]

- Sample incoming direction from some distribution (e.g. proportional to BRDF):
  \[
  \omega_i \sim p(\omega)
  \]
- Recursively evaluate incident radiance in this direction
- Monte Carlo estimator:
  \[
  \frac{f_r(\omega_i \rightarrow \omega_o) L_{o,i}(tr(p, \omega_i), -\omega_i) \cos \theta_i}{p(\omega_i)}
  \]
- Careful! Don’t double-count direct illumination from emissive sources in the recursive call...
Path Tracing Pseudocode

\[
\text{EstRadiance}(x, \omega) \\
p = \text{intersectScene}(x, \omega); \\
L += \text{EstimateDirectLighting}(p, -\omega); \quad \text{\textit{// Next slide}} \\
\]

\[
t\text{Prob} = \text{terminationProbability}(p.\text{brdf}, \omega_i); \\
\text{if } (\text{randomFloat01()} > t\text{Prob}) \\
\quad \omega_i, pdf = p.\text{brdf}.\text{sampleDirection}(x, \omega); \\
\quad L += \text{EstRadiance}(p, \omega_i) * p.\text{brdf}(\omega_i, \omega) \\
\quad \quad * \text{dot}(\omega_i, p.N) / (pdf * (1 - t\text{Prob})); \quad \text{\textit{// Recursive est. of indirect L}} \\
\]

\[
\text{if } (\text{not\_recursive}) \\
\quad L += p.\text{emittedLight}(-\omega); \quad \text{\textit{// Zero-bounce emitted light}} \\
\]

return \( L \);
Direct Lighting Pseudocode (single sample)

```
EstimateDirectLighting(x, ω₀)
    L, ωᵢ, pdf = lights.sampleDirection(x, ω₀); // Imp. sampling
    if (scene.shadowIntersection(x, ωᵢ) // Shadow ray
        return 0;
    else
        return L * x.brdf(ωᵢ, ω₀) * dot(ωᵢ, x.N) / pdf;

// Note: only one random sample over all lights.
// Assignment 3-A asks you to, alternatively, loop over
// multiple lights and take multiple samples (later slide)
```
One path per pixel
32 paths per pixel
Implementation Notes
Paths vs Trees

4 eye rays per pixel
16 shadow rays per eye ray
(64 ray traces per pixel)

64 eye rays per pixel
1 shadow ray per eye ray
(64 ray traces per pixel)
64 image samples x 1 light sample, 13.2 seconds
1 image sample x 64 light samples, 7.0 seconds
8 image samples x 8 light samples, 7.7 seconds
Multiple Light Sources

Consider multiple lights in direct lighting estimate

One strategy:

• Loop over all N lights, sum Monte-Carlo estimates for each light

• For each light: compute Monte Carlo estimate with M samples taken with importance sampling

Needs N * M samples

This is what the assignment asks you to implement.
Multiple Light Sources (Single Sample)

Consider random sampling of multiple lights with a single sample

- Randomly choose light i, with probability $P_i$
- Randomly sample over that light’s directions, with probability density $p_L$
- Probability density of choosing sample is $(P_i \cdot p_L)$
- Weight the lighting calculation by $1/(P_i \cdot p_L)$
- How would you importance sample intelligently?

Can of course average N such samples
Ideal Specular Materials – Issue

Direct illumination problem

• When sampling light randomly, we have zero probability of matching exact direction of the mirror reflection / specular refraction

Remedy

• Importance sample specular BRDFs by generating a sample ray directly along the specular reflection / refraction direction

• See if it hits the light
Numerical Precision Issues

Consider calculating ray-intersection with a distant sphere.

\[ C = (1930.420, 1973.505), \ R = 1 \]
Numerical Precision Issues

\[ C = (1930.420, 1973.505) \quad R = 1 \]

True Intersection: \((1929.7203..., 1972.7897...)\)
Computed Intersection: \((1930.4196..., 1973.5054...)\)
Noisy Shadows

Computed surface intersection

Camera ray

Surface

Shadow ray falsely intersects same surface

Computed surface intersection
Noisy shadows because of floating point precision problems
Floating-Point Precision Remedies

1. double (fp64) rather than float (fp32)
   - 53-bits of precision instead of 24-bits
   - Increase memory footprint

2. Offset origin along ray to ignore close intersections
   - Hard to choose offset that isn’t too small or too big

3. Offset the intersection along the normal
   - Against the normal if the ray is refracted
Offset the intersection along the normal.
Good Scenes for Path Tracing (Diffuse, Sky Lighting)

M. Fajardo, Arnold Path Tracer
Good Scenes for Path Tracing (Diffuse, Sky Lighting)

M. Fajardo, Arnold Path Tracer
Good Scenes for Path Tracing (Diffuse, Sky Lighting)

M. Fajardo, Arnold Path Tracer
Good Scenes for Path Tracing (Diffuse*, Sky Lighting)

Street scene 1
1536x654, 16 paths/pixel, 2 bounces, 250,000 faces, 18 min., dual PIII-800

M. Fajardo, Arnold Path Tracer
A Challenging Scene for Path Tracing – Why?

1000 paths / pixel
Things to Remember

Path tracing

• Partition into direct and indirect illumination
• (Importance) sampling of lighting and BRDF
• Russian Roulette for unbiased finite estimate of infinite series (infinite dimensional integral)
Acknowledgments

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Lecture 18:

Introduction to Material Modeling

Computer Graphics and Imaging
UC Berkeley CS184/284A, Spring 2017
Materials = BRDFs
What is this material?
Diffuse material (BRDF)

Uniform colored diffuse BRDF

Textured diffuse BRDF

[Mitsuba renderer, Wenzel Jakob, 2010]
What is this material?
Glossy material (BRDF)

Copper

Aluminum

[Mitsuba renderer, Wenzel Jakob, 2010]
What is this material?
Ideal reflective / refractive material (BSDF*)

Air <-> water interface

Air <-> glass interface (with absorption)

[Mitsuba renderer, Wenzel Jakob, 2010]
Isotropic / Anisotropic materials (BRDFs)

\[ f_r(\theta_i, \phi_i; \theta_r, \phi_r) \neq f_r(\theta_i, \theta_r, \phi_r - \phi_i) \]
Properties of BRDFs

• Non-negativity

\[ f_{r}(\omega_{i} \to \omega_{r}) \geq 0 \]

• Linearity

\[ L_{r}(p, \omega_{r}) = \int_{H^2} f_{r}(p, \omega_{i} \to \omega_{r}) L_{i}(p, \omega_{i}) \cos \theta_{i} \, d\omega_{i} \]
Properties of BRDFs

- Reciprocity principle

\[ f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \]

- Energy conservation

\[ \forall \omega_r \int_{H^2} f_r(\omega_i \rightarrow \omega_r) \cos \theta_i \, d\omega_i \leq 1 \]
Properties of BRDFs

- **Isotropic vs. anisotropic**

- If isotropic, \( f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i) \)

- Then, from reciprocity,

\[
f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)
\]
Diffuse / Lambertian Material

Light is equally reflected in each output direction

\[ f_r = c \]

Suppose the incident lighting is uniform:

\[
L_o(\omega_o) = \int_{H^2} f_r L_i(\omega_i) \cos \theta_i \, d\omega_i
\]

\[
= f_r L_i \int_{H^2} (\omega_i) \cos \theta_i \, d\omega_i
\]

\[
= \pi f_r L_i
\]

\[
f_r = \frac{\rho}{\pi} \quad \text{— albedo (color)}
\]
Ideal Materials
Perfect Specular Reflection

[Zátonyi Sándor]
Perfect Specular Reflection

\[ \omega_o + \omega_i = 2 \cos \theta \vec{n} = 2(\omega_i \cdot \vec{n})\vec{n} \]

\[ \omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n} \]

\[ \phi_o = (\phi_i + \pi) \mod 2\pi \]

Top-down view (looking down on surface)
Perfect Specular Reflection BRDF

\[ L_i(\theta_i, \phi_i) \quad L_o(\theta_o, \phi_o) \]

\[ L_o(\theta_o, \phi_o) = L_i(\theta_o, \phi_o \pm \pi) \]

\[ f_r(\theta_i, \phi_i; \theta_o, \phi_o) = \frac{\delta(\cos \theta_i - \cos \theta_o)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) \]
Perfect Specular Reflection BRDF
Specular Reflection & the Reflection Equation

\[ L_o(\theta_o, \phi_o) = \int f_r(\theta_i, \phi_i; \theta_o, \phi_o) L_i(\theta_i, \phi_i) \cos \theta_i \, d\cos \theta_i \, d\phi_i \]

\[ = \int \frac{\delta(\cos \theta_i - \cos \theta_o)}{\cos \theta_i} \delta(\phi_i - \phi_o \pm \pi) L_i(\theta_i, \phi_i) \cos \theta_i \, d\cos \theta_i \, d\phi_i \]

\[ = L_i(\theta_r, \phi_r \pm \pi) \]

Whitted’s ray tracing method!
Specular Refraction

In addition to reflecting off surface, light may be transmitted through surface.

Light refracts when it enters a new medium.
Snell’s Law

Transmitted angle depends on
index of refraction (IOR) for incident ray
index of refraction (IOR) for exiting ray

\[
\eta_i \sin \theta_i = \eta_t \sin \theta_t
\]

\[
\varphi_t = \varphi_i \pm \pi
\]

<table>
<thead>
<tr>
<th>Medium</th>
<th>( \eta^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0</td>
</tr>
<tr>
<td>Air (sea level)</td>
<td>1.00029</td>
</tr>
<tr>
<td>Water (20°C)</td>
<td>1.333</td>
</tr>
<tr>
<td>Glass</td>
<td>1.5-1.6</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
</tr>
</tbody>
</table>

* index of refraction is wavelength dependent (these are averages)
Law of Refraction

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

\[
\cos \theta_t = \sqrt{1 - \sin^2 \theta_t} \\
= \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 \sin^2 \theta_i} \\
= \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 (1 - \cos^2 \theta_i)}
\]

Total internal reflection:

When light is moving from a more optically dense medium to a less optically dense medium: \( \frac{\eta_i}{\eta_t} > 1 \)

Light incident on boundary from large enough angle will not exit medium.
Optical Manhole

Total internal reflection
Fresnel Reflection

Reflectance depends on incident angle (and polarization of light)

This example: reflectance increases with grazing angle
Fresnel Reflection (Dielectric, $\eta = 1.5$)
Fresnel Reflectance (Conductor)
Without Fresnel (Fixed Reflectance/Transmission)
Glass with Fresnel Reflection/Transmission
Fresnel Reflection & Transmission
Microfacet Material
Microfacet Reflection

https://twitter.com/Cmdr_Hadfield/status/318986491063828480/photo/1
Microfacet Theory

Rough surface
- Smooth at wavelength scale, rough at microscale
- Flat at macroscale

Individual elements of surface act like mirrors
- Macro BRDF defined by microfacet distribution and local reflection / shadowing
Microfacet BRDF

\[ f(i, o) = \frac{F(i, h)G(i, o, h)D(h)}{4(n, i)(n, o)} \]
Microfacet BRDF: Examples

[Autodesk Fusion 360]

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Anisotropic Reflection

Reflection depends on azimuthal angle $\phi$

Results from oriented microstructure of surface, e.g., brushed metal
Simulation: Nylon

Anisotropic BRDFs are everywhere

[Westin et al. 1992]
Simulation: Velvet

Anisotropic BRDFs are everywhere

[Westin et al. 1992]
Measuring BRDFs
Measuring BRDFs: Motivation

Avoid need to develop / derive models

• Automatically includes all of the scattering effects present

Can accurately render with real-world materials

• Useful for product design, special effects, ...

Theory vs. practice:

[Bagher et al. 2012]
Image-Based BRDF Measurement

Test sample

Light source

Camera positions

Camera

[Marschner et al. 1999]
Measuring BRDFs: gonioreflectometer

Spherical gantry at UCSD
Measuring BRDFs

General approach:

```plaintext
foreach outgoing direction wo
  move light to illuminate surface with a thin beam from wo
for each incoming direction wi
  move sensor to be at direction wi from surface
measure incident radiance
```

Improving efficiency:

- Isotropic surfaces reduce dimensionality from 4D to 3D
- Reciprocity reduces # of measurements by half
- Clever optical systems...
Challenges in Measuring BRDFs

• Accurate measurements at grazing angles
  • Important due to Fresnel effects
• Measuring with dense enough sampling to capture high frequency specularities
• Retro-reflection
• Spatially-varying reflectance, ...
Representing Measured BRDFs

Desirable qualities

• Compact representation
• Accurate representation of measured data
• Efficient evaluation for arbitrary pairs of directions
• Good distributions available for importance sampling
Tabular Representation

Store regularly-spaced samples in \((\theta_i, \theta_o, |\phi_i - \phi_o|)\)

- Better: reparameterize angles to better match specularities

Generally need to resample measured values to table

Very high storage requirements

MERL BRDF Database
[Matusik et al. 2004]
90*90*180 measurements

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Things to Remember

Materials (BRDFs)

• Diffuse, Glossy, ideal specular
• Fresnel
• Microfacet model
• Measured BRDFs
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