Cameras & Lenses Lectures

Last time:
• Field of view, composition & perspective
• Shutter / exposure durations

Today
• Thin lens approximation, Gauss ray diagrams
• Lens focus effects: defocus, depth of field, bokeh

Next time:
• Real lens designs and effects
• Lens exposure considerations
Real Lens Designs Are Highly Complex
Real Lens Elements Are Not Ideal – Aberrations

Real plano-convex lens (spherical surface shape). Lens does not converge rays to a point anywhere.

More discussion next lecture
Today: Thin Lens Approximation
Assume all parallel rays entering a lens pass through its focal point.
Lens Focusing – Conjugate Points

- Rays from a point in object space intersect at a point in image space
- These are called **conjugate points**
- We create images focused at different depths by placing a sensor at the conjugate distances
- Question: what is the relationship between the position of a lens’ conjugate points?
Gauss’ Ray Diagrams
Gauss’ Ray Tracing Construction

Object

Parallel Ray

Chief Ray

Focal Ray

Image
Gauss’ Ray Tracing Construction

What is the relationship between conjugate depths $z_o, z_i$?
Gauss’ Ray Tracing Construction

\[
\frac{h_o}{z_o - f} = \frac{h_i}{f} \quad \text{and} \quad \frac{h_o}{f} = \frac{h_i}{z_i - f}
\]
Gauss’ Ray Tracing Construction

\[
\frac{h_o}{z_o - f} = \frac{h_i}{f} \\
\frac{h_o}{h_i} = \frac{z_o - f}{f} \\
\frac{h_o}{h_i} = \frac{f}{z_i - f} \\
\frac{h_o}{z_i - f} = \frac{f}{z_i - f}
\]

Object / image heights
factor out - applies to all rays

\[(z_o - f)(z_i - f) = f^2 \]

\[z_o z_i - (z_o + z_i)f + f^2 = f^2 \]

\[z_o z_i = (z_o + z_i)f \]

\[\frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o} \]

Newtonian Thin Lens Equation

Gaussian Thin Lens Equation
The Thin Lens Equation

\[
\frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o} \]

where:
- \( f \) is the focal length of the lens
- \( z_o \) is the object distance
- \( z_i \) is the image distance
Changing the Focus Distance

\[ \frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o} \]

- To focus on objects at different distances, move the sensor relative to the lens
- For \( z_i < z_o \) the object is larger than the image
- At \( z_i = z_o \) we have 1:1 macro imaging
- For \( z_i > z_o \) the image is larger than the object
- Can’t focus on objects closer than the lens’ focal length
Magnification

\[ m = \frac{h_i}{h_o} = \frac{z_i}{z_o} \]
Magnification Example – Focus at Infinity

\[ \frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o} \]

\[ m = \frac{z_i}{z_o} \]

If focused on a distant mountain

- \( z_o \approx \infty \), so \( z_i = f \)
- sensor at focal point
- magnification \( \approx 0 \)
Magnification Example – Focus at 1:1 Macro

What configuration do we need to achieve a magnification of 1 (i.e. image and object the same size, a.k.a. 1:1 macro)?

- Need $z_i = z_o$, so $z_i = z_o = 2f$ — sensor at twice focal length
- In 1:1 imaging, if the sensor is 36 mm wide, an object 36 mm wide will fill the frame

$$\frac{1}{f} = \frac{1}{z_i} + \frac{1}{z_o}$$

$$m = \frac{z_i}{z_o}$$
Thin Lens Demonstration

http://graphics.stanford.edu/courses/cs178-10/applets/gaussian.html
Thin Lens Demonstration Observations

3D image of object is:

- Compressed in depth for low magnification
- 1:1 in 3D for unit magnification
- Stretched in depth for high magnification
Lenses transform a 3D object to a 3D image; the sensor extracts a 2D slice from that image.

As an object moves linearly (in Z), its image moves non-proportionally (in Z). And vice versa.

As you refocus a camera, the image changes size!
Defocus Blur
Circle of Confusion
Circle of Confusion

Further defocused object

Closer defocused object
Circle of confusion is proportional to the size of the aperture

\[
\frac{C}{A} = \frac{d'}{z_i} = \frac{|z_s - z_i|}{z_i}
\]
Circle of Confusion is Inversely Proportional to F-Stop

\[ C = A \frac{|z_s - z_i|}{z_i} = \frac{f}{N} \frac{|z_s - z_i|}{z_i} \]

R. Berdan, canadiannaturephotographer.com
Definition: F-Number (a.k.a. F-Stop)

• The F-Number of a lens is defined as the focal length divided by the diameter of the aperture

• Common F-stops on real lenses: 1.4, 2, 2.8, 4.0, 5.6, 8, 11, 16, 22, 32

• 1 stop doubles exposure

• An f-stop of 2 is sometimes written f/2, reflecting the fact that the absolute aperture diameter (A) can be computed by dividing focal length (f) by the relative aperture (N).
Example F-Stop Calculations

- \( D = 50 \text{ mm} \)
  - \( f = 100 \text{ mm} \)
  - \( N = \frac{f}{D} = 2 \)

- \( D = 100 \text{ mm} \)
  - \( f = 200 \text{ mm} \)
  - \( N = \frac{f}{D} = 2 \)

- \( D = 100 \text{ mm} \)
  - \( f = 400 \text{ mm} \)
  - \( N = \frac{f}{D} = 4 \)
Circle of Confusion is Inversely Proportional to F-Stop

\[
C = A \frac{|z_s - z_i|}{z_i} = \frac{f}{N} \frac{|z_s - z_i|}{z_i}
\]

R. Berdan, canadiannaturephotographer.com
Circle of Confusion – Example

50mm f/2 lens
Full frame sensor (36x24mm)
Focus: 1 meter
Background: 10 meter
Foreground: 0.3 meter

\[ A = \frac{50mm}{2} = 25mm \]
\[ z_s = \frac{1}{\frac{1}{50} - \frac{1}{1000}} \approx 52.63mm \]

Background: \[ z_i = \frac{1}{\frac{1}{50} - \frac{1}{10,000}} \approx 50.25mm \]
\[ C = A \frac{|z_s - z_i|}{z_i} = 1.18mm \]
~65 pixels on HD TV

Foreground: \[ z_i = \frac{1}{\frac{1}{50} - \frac{1}{300}} \approx 55.56mm \]
\[ C = A \frac{|z_s - z_i|}{z_i} = 3.07mm \]
~167 pixels on HD TV
**Circle of Confusion in Perspective Composition**

- To maintain field of view on subject, increase distance from subject by same factor as focal length (approx).

- What is the increase in background blur?
Circle of Confusion in Perspective Composition

• To maintain field of view on subject, increase distance from subject by same factor as focal length (approx).

• What is the increase in background blur?

For subject at distance \( Z \),
\[
\frac{1}{z_s} = \frac{1}{f} - \frac{1}{Z}
\]
Distant background means \( z_i = f \),
\[
C = \frac{f}{N} \frac{|z_s - z_i|}{z_i} = \frac{f}{N} \frac{1}{1/f - 1/Z - f} = \ldots
\]
\[
= \frac{f}{N} \frac{f}{Z - f}
\]

• If we increase \( Z \) and \( f \) by factor \( K \), circle of confusion \( C \) also increases by \( K \). (F-stop held constant)
As predicted, \( \frac{100\text{mm}}{28\text{mm}} \approx \frac{138\text{px}}{40\text{px}} \), but notice blur is constant relative to background object itself!
Ray Tracing Ideal Thin Lenses
Examples of Renderings with Lens Focus

Pharr and Humphreys
Ray Tracing for Defocus Blur (Thin Lens)

Setup:
• Choose sensor size, lens focal length and aperture size
• Choose depth of subject of interest $z_o$
  • Calculate corresponding depth of sensor $z_i$ from thin lens equation (focusing)
Ray Tracing for Defocus Blur (Thin Lens)

To compute value of pixel at position \( x' \) by Monte Carlo integration:

- Select random points \( x'' \) on lens plane
- Rays pass from point \( x' \) on image plane \( z_i \) through points \( x'' \) on lens
- Each ray passes through conjugate point \( x''' \) on the plane of focus \( z_o \)
  - Can determine \( x''' \) from Gauss’ ray diagram
  - So just trace ray from \( x'' \) to \( x''' \)
- Estimate radiance on rays using path-tracing, and sum over all points \( x'' \)
Examples of Renderings with Lens Focus

Pharr and Humphreys
Example of Rendering with Lens Focus

Credit: Bertrand Benoit. “Sweet Feast,” 2009. [Blender /VRay]
Example of Rendering with Lens Focus

Credit: Giuseppe Albergo. “Colibri” [Blender]
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