Lecture 24:
Introduction to Color Science (Cont)
Color Reproduction (Cont)
Additive Color Matching Experiment

Show test light spectrum on left
Mix “primaries” on right until they match
The primaries need not be RGB
CIE RGB Color Matching Experiment

Same setup as additive color matching before, but primaries are monochromatic light (single wavelength) of the following wavelengths defined by CIE standard:

- 700 nm
- 546.1 nm
- 435.8 nm

The test light is also a monochromatic light
CIE RGB Color Matching Functions

Graph plots how much of each CIE RGB primary light must be combined to match a monochromatic light
Color Reproduction with Matching Functions

For any spectrum $\Phi(\lambda)$, the perceived color is matched by the following formulas for scaling the CIE RGB primaries

$$c_r = \int_{\lambda} \Phi(\lambda)\overline{r}(\lambda) \, d\lambda$$

$$c_g = \int_{\lambda} \Phi(\lambda)\overline{g}(\lambda) \, d\lambda$$

$$c_b = \int_{\lambda} \Phi(\lambda)\overline{b}(\lambda) \, d\lambda$$
Color Reproduction with Matching Functions

- Given color matching functions in matrix form and new light

\[ C = \begin{pmatrix} \bar{r}(\lambda_1) & \cdots & \bar{r}(\lambda_N) \\ \bar{g}(\lambda_1) & \cdots & \bar{g}(\lambda_N) \\ \bar{b}(\lambda_1) & \cdots & \bar{b}(\lambda_N) \end{pmatrix} \]

\[ \Phi = \begin{pmatrix} \phi(\lambda_1) \\ \vdots \\ \phi(\lambda_N) \end{pmatrix} \]

- Amount of each primary necessary to match is given by

\[ C'\Phi \]
Color Reproduction as Linear Algebra

The projection onto the three response functions can be written in matrix form:

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix} = \begin{bmatrix}
- r_S \\
- r_M \\
- r_L
\end{bmatrix} \begin{bmatrix}
s
\end{bmatrix}
\]

or,

\[V = M_{SML} s.\]
Color Reproduction as Linear Algebra

The spectrum that is produced by the monitor for the color signals R, G, and B is:

\[ s_{a}(\lambda) = R s_{r}(\lambda) + G s_{g}(\lambda) + B s_{b}(\lambda). \]

The discrete form can be written as a matrix:

\[
\begin{bmatrix}
  s_{a} \\
  R \\
  G \\
  B
\end{bmatrix} =
\begin{bmatrix}
  s_{R} & s_{G} & s_{B}
\end{bmatrix}
\begin{bmatrix}
  R \\
  G \\
  B
\end{bmatrix}
\]

or,

\[ s_{a} = M_{RGB} C. \]
What color do we see when we look at the display?

- Feed C to display
- Display produces $s_a$
- Eye looks at $s_a$ and produces $V$

\[
V = M_{SMLM}M_{RGBC}
\]

\[
\begin{bmatrix}
S \\
M \\
L
\end{bmatrix} =
\begin{bmatrix}
r_S s_R & r_S s_G & r_S s_B \\
r_M s_R & r_M s_G & r_M s_B \\
r_L s_R & r_L s_G & r_L s_B \\
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]
Color Reproduction as Linear Algebra

Goal of reproduction: visual response to $s$ and $s_a$ is the same:

$$M_{SML} \tilde{s} = M_{SML} \tilde{s}_a.$$ 

Substituting in the expression for $s_a$:

$$M_{SML} \tilde{s} = M_{SML} M_{RGB} C$$

$$C = (M_{SML} M_{RGB})^{-1} M_{SML} \tilde{s}$$

color matching matrix for RGB
Color Representation
Color Spaces

Need three numbers to specify a color
• but what three numbers?
• a color space is an answer to this question

Common example: monitor RGB
• define colors by what r, g, b signals will produce them on your monitor
  • (in math, \( s = rR + gG + bB \) for some spectra R, G, B)
• device dependent (depends on gamma, phosphors, gains, …)
  • therefore if I choose RGB by looking at my monitor and send it to you, you may not see the same color
• also leaves out some colors (limited gamut), e.g. vivid yellow
Standard Color Spaces

Standardized RGB (sRGB)

- makes a particular monitor RGB standard
- other color devices simulate that monitor by calibration
- sRGB is usable as an interchange space; widely adopted today
- gamut is still limited
A Universal Color Space: CIE XYZ

Imaginary set of standard color primaries X, Y, Z

Designed such that

- X, Y, Z span all observable colors
- Matching functions are strictly positive
- Y is luminance (brightness absent color)

Imaginary because can only be realized with primaries that are negative at some wavelengths
Luminance (Lightness)

Integral of radiance scaled by the visual luminous efficiency

\[ Y = \int \Phi(\lambda) V(\lambda) \, d\lambda \]

Luminous efficiency \( V(\lambda) \) is a measure of how bright a light at a given wavelength is perceived by a human.

https://upload.wikimedia.org/wikipedia/commons/a/a0/Luminosity.png
Separating Luminance, Chromaticity

Luminance: $Y$

Chromaticity: $x, y, z$, defined as

$$x = \frac{X}{X + Y + Z}$$

$$y = \frac{Y}{X + Y + Z}$$

$$z = \frac{Z}{X + Y + Z}$$

- since $x + y + z = 1$, we only need to record two of the three
- usually choose $x$ and $y$, leading to $(x, y, Y)$ coords
CIE Chromaticity Diagram

- Pure (saturated) spectral colors around the edge of the plot.
- Less pure (desaturated) colors in the interior of the plot.
- White at the centroid of the plot (1/3, 1/3).
Gamut

Gamut is the set of chromaticities generated by a set of primaries.

Because definition of xy is linear, interpolation between chromaticities on a chromaticity plot is also linear.

So the gamut is the convex hull of the primary chromaticities.
Gamut

sRGB is a common color space used throughout the internet.

CIE RGB are the monochromatic primaries used for color matching tests described earlier.

sRGB uses ITU-R BT.709 primaries
Red  Green  Blue  White
x   0.64  0.30  0.15  0.3127
y   0.33  0.60  0.06  0.3290
AdobeRGB(98) uses Red and Blue like sRGB and Green like NTSC
CIE-RGB are the primaries for color matching tests: 700/546.1/435.8 nm

Wavelengths in nm
Perceptually Organized Color Spaces
HSV Color Space (Hue-Saturation-Value)

Axes correspond to artistic characteristics of color
Perceptual Dimensions of Color

Hue
- the “kind” of color, regardless of attributes
- colorimetric correlate: dominant wavelength
- artist’s correlate: the chosen pigment color

Saturation
- the “colorfulness”
- colorimetric correlate: purity
- artist’s correlate: fraction of paint from the colored tube

Lightness (or value)
- the overall amount of light
- colorimetric correlate: luminance
- artist’s correlate: tints are lighter, shades are darker
Perceptual Non-Uniformity

• In the xy chromaticity diagram at left, MacAdam ellipses show regions of perceptually equivalent color (ellipses enlarged 10x)

• Must non-linearly warp the diagram to achieve uniform perceptual distances
CIELAB Space (AKA L*a*b*)

A commonly used color space that strives for perceptual uniformity

- L* is lightness
- a* and b* are color-opponent pairs
  - a* is red-green, and b* is blue-yellow
- A gamma transform is used for warping because perceived brightness is proportional to scene intensity $\gamma$, where $\gamma \approx 1/3$
Opponent Color Theory

There’s a good neurological basis for the color space dimensions in CIE LAB

• the brain seems to encode color early on using three axes:
  • white — black, red — green, yellow — blue
  • the white — black axis is lightness; the others determine hue and saturation
  • one piece of evidence: you can have a light green, a dark green, a yellow-green, or a blue-green, but you can’t have a reddish green (just doesn’t make sense)
  • thus red is the opponent to green
  • another piece of evidence: afterimages (following slides)
Even simple judgments – such as lightness depend on brain processing (Anderson and Winawer, Nature, 2005)
Everything is Relative

Edward H. Adelson
Everything is Relative
Everything is Relative
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Things to Remember

Physics of Light
- Spectral power distribution (SPD)
- Superposition (linearity)

Tristimulus theory of color
- Spectral response of human cone cells (S, M, L)
- Metamers - different SPDs with the same perceived color
- Color matching experiment, per-wavelength matching functions
- Color reproduction by integrating color matching functions against input spectrum

Color spaces
- CIE RGB, XYZ, xy chromaticity, LAB, HSV
- Gamut
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CALVIN AND Hobbes

Wow, honey, you're missing a beautiful sunset out here.

I'll count to 10, and then... POK!

Pad, how come old photographs are always black and white? Didn't they have color film back then?

Sure they did, in fact, those old photographs are in color. It's just the world was black and white then.

Really?

Yep. The world didn't turn color until sometime in the 1930s, and it was pretty grainy color for a while.

That's really weird.

Well, truth is stranger than fiction.

But then why are old paintings in color? If the world was black and white, wouldn't artists have painted it that way?

Not necessarily. A lot of great artists were insane.

But... but how could they have painted in color anyway? Wouldn't their paints have been shades of gray back then?

Of course. But they turned colors like everything else did in the '30s.

So why didn't old black and white photos turn color too?

Because they were color pictures of black and white. Remember?

The world is a complicated place, Hobbes.

Whenever it seems that way, I take a nap in a tree and wait for dinner.